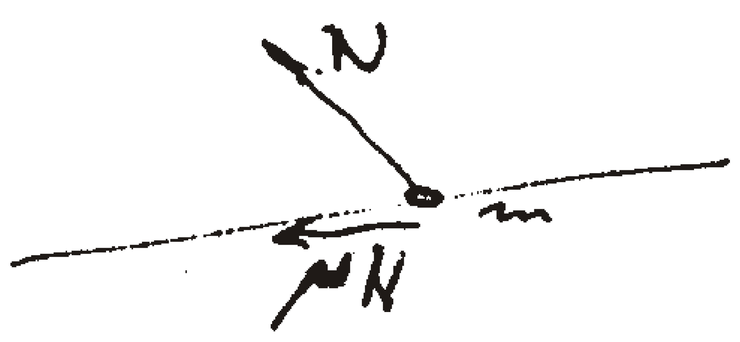
MECANICA NEWTONIANA.

1) a) $\vec{a} = (\ddot{r} - r\omega^2)\,\vec{e_r} + 2\dot{r}\omega\,\vec{e_\varphi}$



$\vec{F} = -\mu N\,\vec{e_r} + N\,\vec{e_\varphi}$

$m(\ddot{r} - r\omega^2) = -\mu\, 2\dot{r}\omega m$

b) $\boxed{\ddot{r} + 2\mu\omega\dot{r} - \omega^2 r = 0}$

$r = A e^{+\lambda_1 t} + B e^{+\lambda_2 t}$, $\lambda_1 = (-\mu + \sqrt{\mu^2+1})\,\omega$

$\lambda_2 = (-\mu - \sqrt{\mu^2+1})\,\omega$

$r(0) = r_0$ $\qquad A + B = r_0$

$\dot{r}(0) = v_0$ $\qquad \lambda_1 A + \lambda_2 B = v_0 = r_0\lambda_1\omega$

$A = r_0$, $B = 0$ $\qquad r = r_0\, e^{(-\mu + \sqrt{\mu^2+1})t}$

$\dot{r} = \lambda_1 r$ $\Rightarrow$ Cuando sale del tubo $\vec{V}_R = \lambda_1 R\,\vec{e_r}$

$\vec{V}_T = R\omega\,\vec{e_\varphi}$ $\Rightarrow$ $\vec{V}_A = \lambda_1 R\,\vec{e_r} + R\omega\,\vec{e_\varphi}$

c) $W = \int_{t_i}^{t_f} \vec{N}\cdot\vec{V}\,dt$ $\qquad \vec{N} = 2m\dot{r}\omega\,\vec{e_\varphi}$

$\vec{V}\cdot\vec{e_\varphi} = r\omega$

$W = \int_{t_i}^{t_f} 2m\dot{r}r\omega^2\,dt = mR^2\omega^2 - mr_0^2\omega^2$

d) $W' = \Delta T = \frac{1}{2}m(\lambda_1^2 R^2 + R^2\omega^2) - \frac{1}{2}m(v_0^2 + r_0^2\omega^2)$

2) a) $-\frac{mv^2}{r} = f(r)$, $\ell = mvr \Rightarrow f(r) = -\frac{\ell^2}{mr^3}$

b) Usando Binet. , $u(\varphi) = 1/r$.

$$\vec{a} = -\frac{\ell^2}{m^2} u^2 (u + u'') \vec{e_r} \qquad \vec{F} = -k u^3 \vec{e_r}$$

$$\frac{\ell^2}{m} u^2 (u + u'') = k u^3$$

$$\frac{\ell^2}{m} u'' - \left(k - \frac{\ell^2}{m}\right) u = 0$$

$$u'' + \gamma u = 0 \qquad \gamma = 1 - \frac{km}{\ell^2}$$

Si $\gamma > 0$ $\quad u = A \cos\sqrt{\gamma}\,\varphi + B \operatorname{sen}\sqrt{\gamma}\,\varphi$, A y B dependen de las condiciones iniciales.

Si $\gamma = 0$ $\quad u = A\varphi + B$

Si $\gamma < 0$ $\quad u = A \cosh\sqrt{-\gamma}\,\varphi + B \operatorname{senh}\sqrt{-\gamma}\,\varphi$.

c) $\ell = m v_0 r_0$, $\gamma = 1 - \frac{k}{m v_0^2 r_0^2}$,

$\gamma > 0$ si $m v_0^2 r_0^2 < k$ o sea $|v_0| < \sqrt{\frac{k}{m r_0^2}}$.

Si $|v_0| > \sqrt{\frac{k}{m r_0^2}}$ $(\gamma > 0)$ $u = \frac{1}{r_0} \cos\sqrt{\gamma}\,\varphi$

Si $|v_0| = \sqrt{\frac{k}{m r_0^2}}$ $(\gamma = 0)$ $r = r_0$.

Si $|v_0| < \sqrt{\frac{k}{m r_0^2}}$ $(\gamma < 0)$ $u = \frac{1}{r_0} \cosh\sqrt{-\gamma}\,\varphi$