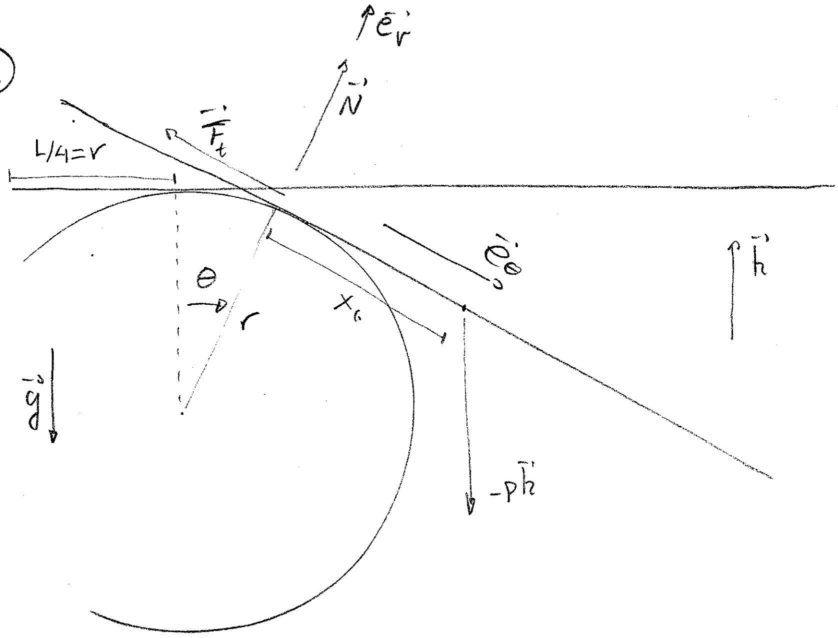
① 

ⓐ

$$x_G = \frac{L}{2} - \left(\frac{L}{4} + r\theta\right)$$

$$x_G = \frac{L}{4} - r\theta = r(1-\theta)$$

$$\vec{r}_G = r\vec{e}_r + x_G\,\vec{e}_\theta$$

$$\vec{v}_G = r\dot{\vec{e}}_r + \dot{x}_G\,\vec{e}_\theta + x_G\,\dot{\vec{e}}_\theta =$$

$$= r\dot{\theta}\,\vec{e}_\theta - r\dot{\theta}\,\vec{e}_\theta + x_G(-\dot{\theta}\vec{e}_r) =$$

$$= -r(1-\theta)\dot{\theta}\,\vec{e}_r$$

ⓑ $E_{mec.} = cte$ $\qquad T + U = E_{mec.}$

$$U = mg(\vec{r}_G\cdot\vec{k}) = mg\left(r\,\vec{e}_r\cdot\vec{k} + (r - r\theta)\underbrace{\vec{e}_\theta\cdot\vec{k}}_{-\operatorname{sen}\theta}\right) =$$

$$= mg\left(r\cos\theta - (r - r\theta)\operatorname{sen}\theta\right)$$

$$T = \frac{1}{2}m v_G^2 + \frac{1}{2}I_G\dot{\theta}^2 \quad ; \quad I_G = \frac{mL^2}{12}$$

$$T = \frac{1}{2}m(r - r\theta)^2\dot{\theta}^2 + \frac{2mr^2}{3}\dot{\theta}^2 = \frac{mr^2}{2}(1-\theta)^2\dot{\theta}^2 + \frac{2}{3}mr^2\dot{\theta}^2$$

$$T + U = mgr \quad (E_{mec}(t=0)) \quad \Rightarrow \quad T + U - mgr = 0$$

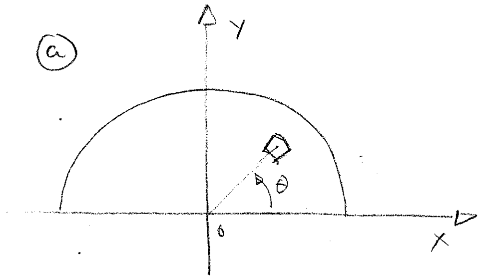
$$\boxed{mr^2\dot{\theta}^2\left[\frac{(1-\theta)^2}{2} + \frac{2}{3}\right] + mgr\left(\cos\theta - 1 - (1-\theta)\operatorname{sen}\theta\right) = 0.}$$

ⓒ $m\vec{a}_G = N\vec{e}_r - F_t\vec{e}_\theta - mg\vec{k}$

$$\vec{a}_G = \left(-(r - r\theta)\ddot{\theta} + r\dot{\theta}^2\right)\vec{e}_r - (r - r\theta)\dot{\theta}^2\,\vec{e}_\theta$$

$$\begin{cases} N = mg\cos\theta + mr\dot{\theta}^2 - m(r - r\theta)\ddot{\theta} \\ F_t = mg\operatorname{sen}\theta + m(r - r\theta)\dot{\theta}^2 \end{cases}$$

(2) (a)

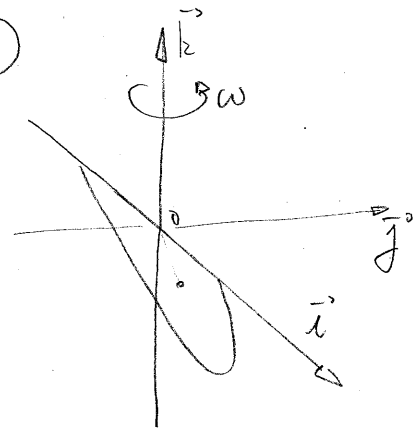


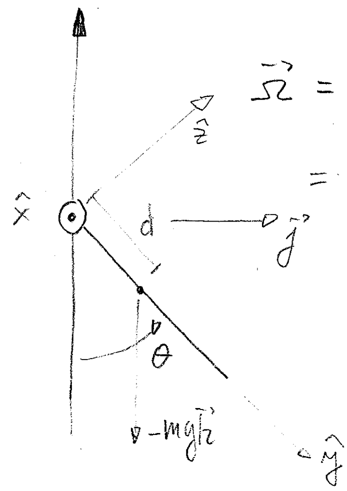
$$Y_G = \frac{1}{m}\int_0^{\pi}\int_0^{R} \sigma r \operatorname{sen}\theta \, r \, d\theta \, dr = \frac{\sigma}{m}\int_0^{\pi}\int_0^{R} r^2 \operatorname{sen}\theta \, dr \, d\theta =$$

$$= \frac{\sigma}{m}\cdot\frac{R^3}{3}\cdot 2 \quad ; \quad m = \frac{\sigma R^2 \pi}{2} \Rightarrow Y_G = \frac{4R}{3\pi}$$

$$\underset{(x,y,z)}{\mathbb{I}_0} = \frac{mR^2}{4}\begin{bmatrix} 1 & & \\ & 1 & \\ & & 2 \end{bmatrix}$$

(b)





$$\vec{\Omega} = \omega\vec{k} + \dot{\theta}\,\vec{i} =$$

$$= -\omega\cos\theta\,\hat{y} + \omega\operatorname{sen}\theta\,\hat{z} + \dot{\theta}\,\hat{x}$$

$$\mathbb{I}_0\vec{\Omega} = \frac{mR^2}{4}\left[\dot{\theta}\,\hat{x} - \omega\cos\theta\,\hat{y} + 2\omega\operatorname{sen}\theta\,\hat{z}\right]$$

$$\frac{d}{dt}\left(\mathbb{I}_0\vec{\Omega}\right) = \frac{mR^2}{4}\Big(\ddot{\theta}\,\hat{x} + \dot{\theta}\,\dot{\hat{x}} + \omega\operatorname{sen}\theta\,\dot{\theta}\,\hat{y} - \omega\cos\theta\,\dot{\hat{y}} + 2\omega\cos\theta\,\dot{\theta}\,\hat{z} +$$

$$+ 2\omega\operatorname{sen}\theta\,\dot{\hat{z}}\Big) = \frac{mR^2}{4}\left(\ddot{\theta} + \omega^2\operatorname{sen}\theta\cos\theta - 2\omega^2\operatorname{sen}\theta\cos\theta\right)\hat{x} +$$

$$\dot{\hat{x}} = \hat{\Omega}\times\hat{x} = \omega\,\vec{j}$$

$$\dot{\hat{y}} = \hat{\Omega}\times\hat{y} = -\omega\operatorname{sen}\theta\,\hat{x} + \dot{\theta}\,\hat{z}$$

$$\dot{\hat{z}} = \hat{\Omega}\times\hat{z} = -\omega\cos\theta\,\hat{x} - \dot{\theta}\,\hat{y}$$

$$+ \frac{mR^2}{4}\Big(\dot{\theta}\omega\operatorname{sen}\theta + \omega\operatorname{sen}\theta\,\dot{\theta} -$$

$$- 2\omega\operatorname{sen}\theta\,\dot{\theta}\Big)\,\hat{y} +$$

$$+ \frac{mR^2}{4}\Big(\dot{\theta}\omega\cos\theta - \omega\cos\theta\,\dot{\theta} +$$

$$+ 2\omega\cos\theta\,\dot{\theta}\Big)\,\hat{z}$$

$$\frac{d}{dt}\left(\mathbb{I}_0\vec{\Omega}\right) = \vec{M}^{ext}_0$$

$$\frac{d}{dt}\left(\mathbb{I}_o \vec{\Omega}\right)\cdot\vec{i} = \vec{M}^{ext}_o\cdot\vec{i} = -mg\,d\,\text{sen}\theta$$

$$\parallel$$

$$\frac{mR^2}{4}\left(\ddot{\theta} - \omega^2\text{sen}\theta\cos\theta\right) = -mg\frac{4R}{3\pi}\text{sen}\theta$$

$$\boxed{\ddot{\theta} - \omega^2\text{sen}\theta\cos\theta + \frac{16g}{3\pi R}\text{sen}\theta = 0}$$

(c)

$$\vec{M}^{ext,react}_o = \frac{mR^2}{2}\omega\dot{\theta}\cos\theta\,\hat{z}$$

$$\dot{\theta}\ddot{\theta} = \omega^2\text{sen}\theta\cos\theta\,\dot{\theta} - \frac{16g}{3\pi R}\text{sen}\theta\,\dot{\theta}$$

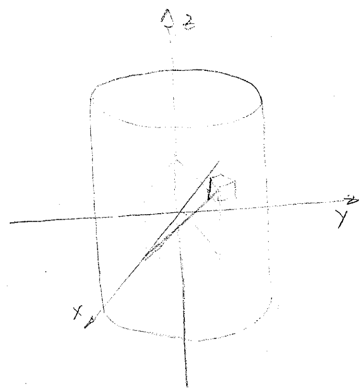
$$\frac{\dot{\theta}^2}{2} = \omega^2\left(\frac{\text{sen}^2\theta}{2} - \frac{\text{sen}^2\theta_m}{2}\right) + \frac{16g}{3\pi R}\left(\cos\theta - \cos\theta_m\right)$$

$$\vec{M}^{ext,react}_o = \hat{z}\,\frac{mR^2}{2}\omega\cos\theta\sqrt{\omega^2(\text{sen}^2\theta - \text{sen}^2\theta_m) + \frac{32g}{3\pi R}(\cos\theta - \cos\theta_m)}$$

3

(a)

$I_{0,z} = \frac{mR^2}{2}$



$$I_{x,0} = \int_{-h}^{h}\int_0^R\int_0^{2\pi} \rho\, r\,d\theta\,dr\,dz\,(z^2 + r^2\,\text{sen}^2\theta) =$$

$$= \rho \int_{-h}^{h}\int_0^R\int_0^{2\pi} (z^2 r + r^3\, \text{sen}^2\theta)\,dr\,d\theta\,dz =$$

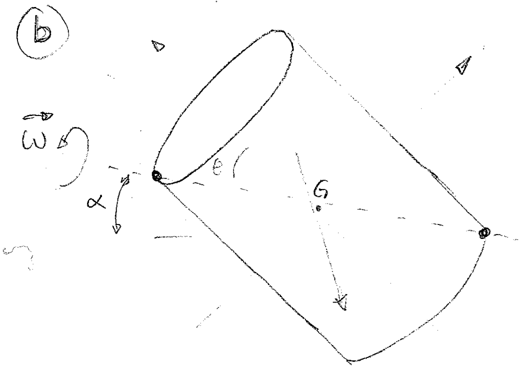
$$= \rho \int_0^{2\pi}\int_{-h}^{h} z^2\frac{R^2}{2} + \frac{R^4}{4}\,\text{sen}^2\theta\, dz\,d\theta = \rho\int_0^{2\pi} \frac{h^3R^2}{3} + \frac{hR^4}{2}\,\text{sen}^2\theta\,d\theta =$$

$$= \rho\left(\frac{h^3R^2}{3}\cdot 2\pi + \frac{hR^4}{2}\pi\right) = m\frac{h^2}{3} + m\frac{R^2}{4} = m\left(\frac{h^2}{3}+\frac{R^2}{4}\right)$$

$m = \rho R^2\pi 2h$

$$\mathbb{I}_{O(x,y,z)} = \begin{bmatrix} m\left(h^2/3 + R^2/4\right) & & \\ & m\left(h^2/3 + R^2/4\right) & \\ & & mR^2/2 \end{bmatrix}$$

(b)



$$\frac{d}{dt}\left(\mathbb{I}_G \vec\omega\right) = \vec M^{ext}_G$$

$$\vec\omega = \omega\cos\theta\,\hat z - \omega\,\text{sen}\,\theta\,\hat y$$

$$\dot{\vec\omega} = \overset{=0}{\dot\omega}\cos\theta\,\hat z + \omega\cos\theta\,\dot{\hat z} - \overset{=0}{\dot\omega}\,\text{sen}\,\theta\,\hat y - \omega\,\text{sen}\,\theta\,\dot{\hat y} =$$

$$= \overset{=0}{\dot\omega}\cos\theta\,\hat z - \dot\omega\,\text{sen}\,\theta\,\hat y - \omega\cos\theta + \omega\,\text{sen}\,\theta)\,\hat x$$

$$\dot{\hat y} = \vec\omega\times\hat y = -\omega\cos\theta\,\hat x$$

$$\dot{\hat z} = \vec\omega\times\hat z = -\omega\,\text{sen}\,\theta\,\hat x$$

$$\mathbb{I}_G\vec\omega = -I_y\,\omega\,\text{sen}\,\theta\,\hat y + I_z\,\omega\cos\theta\,\hat z$$

$$\frac{d}{dt}\left(\mathbb{I}_G\vec\omega\right) = -I_y\overset{=0}{\dot\omega}\,\text{sen}\,\theta\,\hat y - I_y\,\omega\,\text{sen}\,\theta\,\dot{\hat y} + I_z\overset{=0}{\dot\omega}\cos\theta\,\hat z + I_z\,\omega\cos\theta\,\dot{\hat z} =$$

$$= -I_y\overset{=0}{\dot\omega}\,\text{sen}\,\theta\,\hat y + I_y\,\omega^2\,\text{sen}\,\theta\cos\theta\,\hat x + I_z\overset{=0}{\dot\omega}\cos\theta\,\hat z - I_z\,\omega^2\,\text{sen}\,\theta\cos\theta\,\hat x$$

$$\forall\theta\quad \omega^2\left(m\left(\frac{h^2}{3}+\frac{R^2}{4}\right) - m\frac{R^2}{2}\right)\text{sen}\,\theta\cos\theta = 0 \Rightarrow \frac{h^2}{3} = \frac{R^2}{4} \Rightarrow \boxed{h = \frac{R\sqrt{3}}{2}}$$